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Compose Walsh Sequences

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Orthogonal Sequences

The set **S** $= \{X = (x_1, x_2, \dots, x_n); x_i \in F_2 = \{0,1\}\}$

is Orthogonal iff:

1. For each X in **S**

$$\left| \text{Number of "1"s} - \text{Number of "0"s} \right| \leq 1$$

2. For each X, Y in **S** and $X \neq Y$ then in X+Y :

$$\left| \text{Number of "1"s} - \text{Number of "0"s} \right| \leq 1$$

Addition by "*mod* 2"

The Rules of Addition are:

$$
\begin{array}{r}
0 \\
+ \\
0 \\
\hline
0
\end{array}
\qquad
\begin{array}{r}
1 \\
+ \\
0 \\
\hline
1
\end{array}
\qquad
\begin{array}{r}
0 \\
+ \\
1 \\
\hline
1
\end{array}
\qquad
\begin{array}{r}
1 \\
+ \\
1 \\
\hline
0
\end{array}
$$

Example

$$
\begin{array}{ccccccc}
1 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 \\
\hline
1 & 1 & 0 & 0 & 1 & 1 & 1
\end{array}
\; +
$$

Walsh's Sequences W of order 2^k

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Each sequence w of W except the Zero sequence w_0 :

* Has the length 2^k

* Contains 2^{k-1} of "1"s

* Contains 2^{k-1} of "0"s

And W is closed under the Addition

Table1: W_4–Walsh Sequences of Order $2^2 = 4$

$w_0 =$	0	0	0	0
$w_1 =$	0	0	1	1
$w_2 =$	0	1	1	0
$w_3 =$	0	1	0	1

Table2: W_8 –Walsh Sequences of Order $2^3 = 8$

$w_0 =$	0	0	0	0	0	0	0	0
$w_1 =$	0	0	0	0	1	1	1	1
$w_2 =$	0	0	1	1	1	1	0	0
$w_3 =$	0	0	1	1	0	0	1	1
$w_4 =$	0	1	1	0	0	1	1	0
$w_5 =$	0	1	1	0	1	0	0	1
$w_6 =$	0	1	0	1	1	0	1	0
$w_7 =$	0	1	0	1	0	1	0	1

Number of “1”s and Number of “0’ in x(w)

x	
No of "1"s	No of "0"s
n_1	n_2

w	
No of "1"s	No of "0"s
m_1	m_2

Then:

* Number of "1"s in x(w) is:

$$n_1 \cdot m_1 + n_2 \cdot m_2$$

* Number of "0"s in x(w) is:

$$n_1 \cdot m_2 + n_2 \cdot m_1$$

Example for x(w)

$x = 1010110, \quad n_1 = 4, n_2 = 3$

$w = 110, m_1 = 2, m_2 = 1, \bar{w} = 001$

$x(w) = 110\ 001\ 110\ 001\ 110\ 110\ 001$

Number of "1"s in x(w) is:

$n_1 \cdot m_1 + n_2 \cdot m_2 = 4(2)+3(1) = 11$

Number of "0"s in x(w) is:

$n_1 \cdot m_2 + n_2 \cdot m_1 = 4(1)+3(2) = 10$

Walsh Sequences

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* A is Walsh Sequences of Order 2^m

* B is Walsh Sequences of Order 2^n

Compose A with B: $W_k = A(b_k)$; $w_i = a_i(b_k)$

A	
No of "1's	No of "0's
2^{m-1}	2^{m-1}

B	
No of "1's	No of "0's
2^{n-1}	2^{n-1}

* Number of "1"s in W_i is: 2^{m+n-1}
* Number of "0"s in W_i is: 2^{m+n-1}
* The diference between the number of '1" and the number of "0" is zero
* $W_i + W_j$ has the same number of "1" and the same number of "0" and the same difference

Result 1

$W_k = A(b_k)$ is Orthogonal set

Compose B with A: $\overline{W}_k = B(a_k)$; $w_i = b_i(a_k)$

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B		A	
No of "1's	No of "0's	No of "1's	No of "0's
2^{n-1}	2^{n-1}	2^{m-1}	2^{m-1}

* Number of "1"s in $\overline{W}_i$ is: 2^{n+m-1}
* Number of "0"s in $\overline{W}_i$ is: 2^{n+m-1}
* The diference between the number of '1" and the number of "0" is zero
* $\overline{W}_i + \overline{W}_j$ has the same number of "1" and the same number of "0" and the same difference

Result 2

$\bar{W}_k = \mathbf{B}(\mathbf{a}_k)$ **is Orthogonal set**

Example 1: $A = W_4^*$, $B = W_8^*$, A(B)

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$a_1(b_1) = (11110000\ 11110000\ 00001111\ 00001111)$

$a_2(b_1) = (11110000\ 00001111\ 00001111\ 11110000)$

$a_3(b_1) = (11110000\ 00001111\ 11110000\ 00001111)$

* The length is: $2^{2+3} = 2^5 = 32$
* Number of "1"s = Number of "0" = $2^{2+3-1} = 2^4 = 16$

Example2: $B = W_8^*$, $A = W_4^*$, B(A)

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$b_1(a_1)$ = (1100 1100 1100 1100 0011 0011 0011 0011)
$b_2(a_1)$ = (1100 1100 0011 0011 0011 0011 1100 1100)
$b_3(a_1)$ = (1100 1100 0011 0011 1100 1100 0011 0011)
$b_4(a_1)$ = (1100 0011 0011 1100 1100 0011 0011 1100)
$b_5(a_1)$ = (1100 0011 0011 1100 0011 1100 1100 0011)
$b_6(a_1)$ = (1100 0011 1100 0011 0011 1100 0011 1100)
$b_7(a_1)$ = (1100 0011 1100 0011 1100 0011 1100 0011)

* The length is: $2^{3+2} = 2^5 = 32$
* Number of "1"s = Number of "0"s $= 2^{3+2-1} = 2^4 = 16$

Conclusion and Results

Result 1. Generate $2^n - 1$ sets of Orthogonal Sequences with length 2^{m+n} and minimum distance 2^{m+n-1}.

Result 2. Generate $2^m - 1$ sets of Orthogonal Sequences with length 2^{m+n} and minimum distance 2^{m+n-1}.

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Thank you for attention