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Compose Walsh Sequences

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Compose Two Sequences

The set $S = \{X = (x_1, x_2, \ldots, x_n); x_i \in F_2 = \{0,1\}\}$

Is Orthogonal iff:

1. For each X in S

   $\left|Number\ of\ "1"_s - Number\ of\ "0"_s\right| \le 1$

2. For each X, Y in S and X≠Y then in X+Y

   $\left|Number\ of\ "1"_s - Number\ of\ "0"_s\right| \le 1$

Compose B with A: $\overline{W}_k = B(a_k)$; $\overline{w}_i = b_i(a_k)$

B		A	
No of "1's	No of "0's	No of "1's	No of "0's
2^{n-1}	2^{n-1}	2^{m-1}	2^{m-1}

- Number of "1"s in w_i is: 2^{n+m-1}
- Number of "0"s in w_i is: 2^{n+m-1}
- The difference between the number of "1"s and the "0"s is zero
- $w_i + w_j$ has the same number of "1"s and the same number of "0"s and the same difference

W_8 - Walsh Sequences of Order $2^3 = 8$

W0 = (0 0 0 0 0 0 0 0)
W2 = (0 0 0 0 1 1 1 1)
W3 = (0 0 1 1 1 1 0 0)
W4 = (0 1 1 0 0 1 1 0)
W5 = (0 1 1 0 1 0 0 1)
W6 = (0 1 0 1 1 0 1 0)
W7 = (0 1 0 1 0 1 0 1)

Example 1: $A = W_4^*$, $B = W_8^*$, A(B)

b_1 = (0 0 0 0 1 1 1) thus b_1 = (1 1 1 1 0 0 0 0)

- $a_1(b_1)$=(1111000 11110000 00001111 00001111)
- $a_2(b_2)$=(1111000 11110000 00001111 00001111)
- $a_3(b_3)$=(1111000 11110000 00001111 00001111)

- The length is: $2^{2+3} = 2^5 = 32$
- Number of "1"s = Number of "0"s = $2^{2+3-1} = 16$

Number of "1"s and Number of "0"s in x(w)

X		W	
No of "1's	No of "0's	No of "1's	No of "0's
n_1	n_2	m_1	m_2

Then:

1. Number of "1"s in x(w) is: $n_1 m_1 + n_2 m_2$
2. Number of "0"s in x(w) is: $n_1 m_2 + n_2 m_1$

Example 2: $B = W_8^*$, $A = W_4^*$, B(A)

a1= (0 0 1 1) thus a1 = (1 1 0 0), Thus B(a)
b1(a1)=(1100 1100 1100 1100 0011 0011 0011 0011)
b2(a1)=(1100 1100 0011 0011 0011 0011 1100 1100)
b3(a1)=(1100 1100 0011 0011 1100 1100 0011 0011)
b4(a1)=(1100 0011 0011 1100 1100 0011 0011 1100)
b5(a1)=(1100 0011 0011 1100 0011 1100 1100 0011)
b6(a1)=(1100 0011 1100 0011 0011 1100 0011 1100)
b7(a1)=(1100 0011 1100 0011 1100 0011 1100 0011)
The length is: 23+2 = 25 =32
Number of "1"s = Number of "0"s =23+2-1=16

Compose A with B: $W_K = A(b_k)$; $w_i = a_i(b_k)$

A		B	
No of "1's	No of "0's	No of "1's	No of "0's
2^{m-1}	2^{m-1}	2^{n-1}	2^{n-1}

1. Number of "1"s in w_i is: 2^{m+n-1}
2. Number of "0"s in w_i is: 2^{m+n-1}

- The difference between the number of "1"s and the "0"s is zero
- $w_i + w_j$ has the same number of "1"s and the same number of "0"s and the same difference

CONCLUSION AND RESULTS

Result 1.

Generate 2^{n-1} sets of Orthogonal Sequences with the length 2^{m+n} and minimum distance 2^{m+n-1}

Result 2.

Generate 2^{m-1} sets of Orthogonal Sequences with the length 2^{m+n} and minimum distance 2^{m+n-1}